P425/1
PURE
MATHEMATICS
2022
PAPER 1
3hrs

MATIGO MOCK EXAMINATIONS 2022
Uganda Advanced Certificate of Education
PURE MATHEMATICS
PAPER 1
3hours

INSTRUCTIONS TO CANDIDATES:

- ***Attempt all the eight questions in Section A and Not more than five from Section B.***
- ***Any additional question(s) will not be marked***
- ***All working must be shown clearly***
- ***Silent non-programmabe calculators and mathematical tables with a list of formulae may be used.***

SECTION A (40MARKS)

Answer **all** questions in this section

1. Solve the equation; $4cos2x + 3sin2x = 3$ for $0^0 \leq x \leq 360^0$. (5marks)

2. Using the binomial theorem, find the quadratic function that approximates to $f(x) = \frac{1}{(1-3x)^{\frac{2}{3}}}$ for values of x close to zero. Hence evaluate $\sqrt[3]{\frac{64}{25}}$ correct to 3 decimal places. (5marks)

3. Given that $y = Intan\left(\theta/_2\right)$, prove that $\frac{d^2y}{d\theta^2} + cot\theta\frac{dy}{d\theta} = 0$. (5marks)

4. The lines L_1 and L_2 are given by the equations; $\frac{x-3}{k} = y - 4 = \frac{z-4}{-k}$ and $\frac{x-8}{1} = \frac{y-1}{3} = \frac{z-3}{3}$ respectively. Find the values of K for which L_1 and L_2 interest and hence find the position vector of the point of intersection.

(5marks)

5. Given that $\beta\alpha^3$ and $\alpha\beta^3$ are the roots of the equation $x^2 + 26x + 16 = 0$. Find the possible equation(s) whose roots are α *and* β. (5marks)

6. Find $\int \frac{2+tan^2x}{x+tanx} dx$. (5marks)

7. Find the equation of the line which is perpendicular to the line $3x + 2y = 1$ and passes through the point of intersection of the lines $x + 2y - 1 = 0$ and $2x - y + 8 = 0$ (5marks)

8. Given that $x = cot\theta$ and $y = cosec\theta$ show that $\frac{d^2x}{dy^2} = -tan^3\theta$ (5marks)

SECTION B (60MARKS)

9. a) Prove by induction that $10^{2n-1} + 1$ is exactly divisible by 11 for $n \geq 1$. (5marks)

b) The polynomial $x^4 + px^3 - x^2 + qx - 12$ leaves no remainder when divided by $x + 1$ and $x + 2$. Find the values of p and q , hence factorise the polynomial completely. (7marks)

10. a) Given that $y = \frac{x(x-3)}{(x-1)(x-4)}$.

i) Show that the curve does not have turning points (4marks)

ii) Find the asymptotes and intercepts on the curve. (4marks)

iii) Hence sketch the curve. (4marks)

11. a) Express $\frac{x^3-3}{(x-2)(x^2+1)}$ into partial fractions. (7marks)

b) Hence find $\int \frac{x^3-3}{(x-2)(x^2+1)} dx$ (5marks)

12. a) Given that $Z = 5 - 2i$, express $Z^* - \frac{3}{Z}$ in form $a + ib$ hence determine its modulus and argument. (6marks)

b) Given that $\left|\frac{3z+1}{2z-i}\right| = \sqrt{2}$, find the locus of z. Hence describe the locus. (6marks)

13. a) The line $\frac{x+1}{2} = \frac{y+3}{a} = \frac{z+2}{3}$ lies in the plane $x + 2y + bz = 3$. Find the values of a and b. (4marks)

b) A line and a plane are given by the equations $\frac{x+1}{2} = \frac{y-3}{5} = \frac{z+1}{-1}$ and $2x - y + 3z = 20$ respectively. Find;

i) the point of intersection of the line and the plane. (4marks)

ii) The acute angle between them. (4marks)

14. Prove that in any triangle ABC, $b + c = acos\left(\frac{B-C}{2}\right) cosec\frac{A}{2}$. (6marks)

b) Solve the equation, $3sin2\theta + 4cos^2\theta = -1$, for $0^0 \leq \theta \leq 180^0$

15. a) Given that $siny = 2six$, show that $\left(\frac{dy}{dx}\right)^2 = 1 + 3sec^2y$ and deduce that $\frac{d^2y}{dx^2} = 3sec^2y\ tany$. (7marks)

b) Find the volume generated when the area between $y = e^x$, the axes and the line $x = 1$ is rotated through one complete revolution about the $x$axis. (5marks)

16. a) Solve the deferential equation; $\frac{dy}{dx} - ytanx = cosx$. (5marks)

b)The rate of cooling of a body in air is said to be proportional to the difference between the temperature θ of the body and the temperature θ_0 of the air. If the temperature of air is kept constant at 20^0C and the body cools from 100^0C to 60^0C in 20minutes in what time will the body cool to 30^0C? (7marks)

END